Freeze-Out, Pauses, and Distances: An Analysis of Reverse Annealing Performance

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Introduction and Motivation

Introduction to Quantum Annealing

- Quantum annealing approximately solves quadratic unconstrained binary optimization (QUBO) problems utilizing the adiabatic principle [1].
- The optimization is performed by sampling the system's lowest-energy state corresponding to the **problem Hamiltonian** H_P.
- The system's total Hamiltonian (Eq. 1) depends on the **annealing schedule** $s(t)$, which controls the transition from the mixing Hamiltonian H_M to H_P .
- The schedule shape and total annealing time strongly influence the dynamics; special cases like **annealing pauses** correspond to flat regions in $s(t)$[2].

The Concept of Reverse Annealing

- **Forward annealing (FA)** starts from a quantum superposition of all qubits and gradually increases the influence of the problem Hamiltonian until the system settles into a classical state representing a solution [1][2].
- **Reverse annealing (RA)** instead begins from a known classical candidate and temporarily reintroduces quantum fluctuations to explore nearby states and refine the solution [3].
- In this case two main variants exist, **repeated** and **iterative** reverse annealing, both start from an explicitly prepared classical state [3].
- In FA, before measurement, the system reaches a **freeze-out** point where dynamics slow and evolution effectively halts. In RA, this point is key, it marks where quantum fluctuations are reactivated to enable local exploration [4].

$$H(t) = A(s(t))H_M + B(s(t))H_P, \quad (1)$$

$$H_M = -\frac{1}{2}\sum_i \sigma_i^x, \quad (2)$$

$$H_P = -\frac{1}{2}\left(\sum_i h_i\sigma_i^z + \sum_{i>j} J_{i,j}\sigma_i^z\sigma_j^z\right). \quad (3)$$

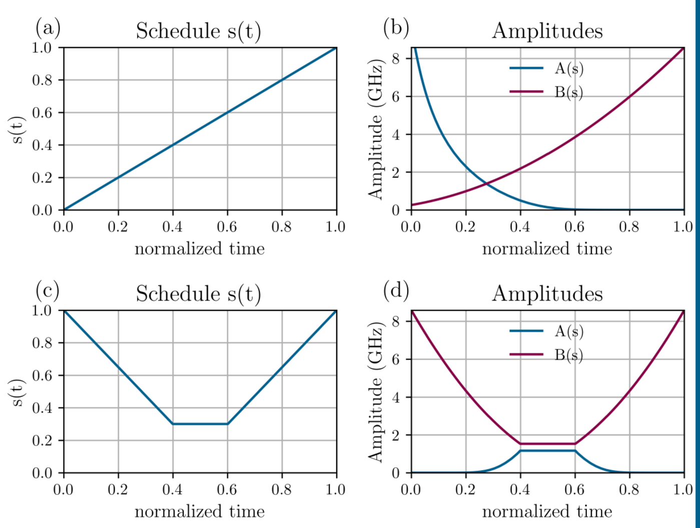


Figure 1: Annealing schedules for FA and RA

Refining Forward using Reverse Annealing

Forward annealing, the standard approach to quantum annealing, effectively solves QUBO problems. However, as problem sizes increase, hardware imperfections such as qubit errors increasingly degrade results, despite careful tuning and long annealing times. This motivates the search for refinement strategies like RA to boost FA performance when it starts to falter. This has been done in several studies [5,6].

Research Questions:

- For which problem classes is RA most effective?
- Which properties determine whether a problem class benefits from RA?
- What annealer configurations yield optimal performance across problem classes?

Reverse Annealing as Refinement

Set of Tested Problems

To assess RA across a broad spectrum of QUBO features, three problem classes were selected, differing in constraints, sparsity, hardness, value distributions, and size.

Max Cut – Baseline [7]

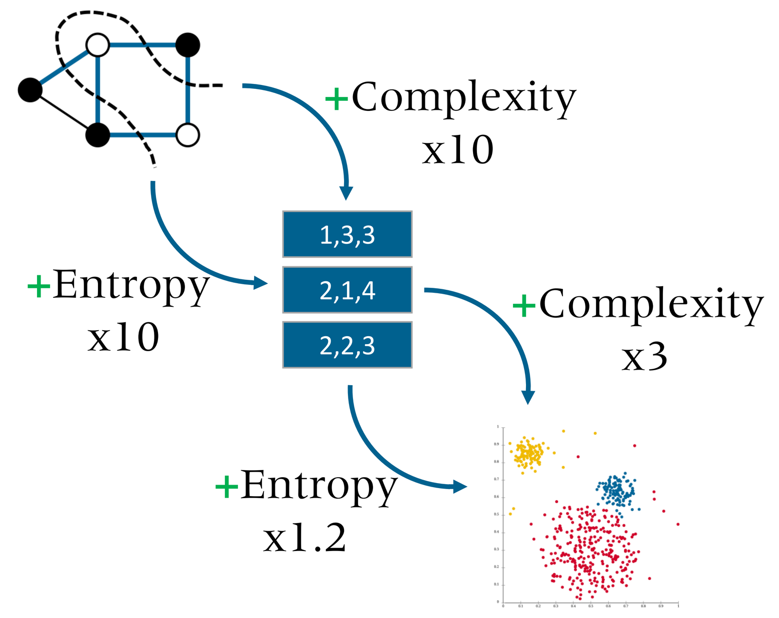
- No constraints
- High sparsity (70% empty)
- Low variation in QUBO entries (Var=0.01)

Number Partition – Dense Problems [8]

- With constraints and encodings (One-Hot)
- Low sparsity (0.1% empty)
- High variation in QUBO entries (Var=0.2)

Sparse Clustering – Complex Problems [9]

- With constraints and encodings (One-Hot)
- QUBO density is decreased (84% empty)
- Largest problems (up to 300 logical qubits)



Exploration vs. Exploitation: Device Settings

RA, like other refinement strategies, must be tuned to balance exploration of new solutions and exploitation of the FA result. This balance is examined by comparing different device settings:

Settings	Effect	Note
↓ Reverse distance ↑ Annealing time	More exploration	Longer time amplifies effect
↑ Reverse distance ↑ Annealing Pause	Steeper slope in $s(t)$	More abrupt quantum transition
↑ Annealing time ↑ Annealing Pause	Longer, smoother pause	Gentler amplitude change

Dynamics and Problem Characteristics

Different problem characteristics influence annealer performance, especially those with physical origins:

Characteristic	Consequence
↑ Problem size	↑ Bit-flip error likelihood
↑ Chain length	↑ Chain breaks → distorted Hamiltonian
↑ QUBO value range	↑ Crosstalk and calibration errors

To study system dynamics beyond solution quality, we approximate freeze-out points via the slicing technique [10]. For each problem class and size, one representative instance was tested at $100\,\mu s$ annealing time across schedule points $s = [0.2, 0.3, 0.4, 0.5, 0.6, 0.7]$.

Experimental Results

Reverse Annealing Performances

- **Baseline:** Forward Annealing with tuned chain strengths per problem class, run at $20\,\mu s$, $100\,\mu s$, and $500\,\mu s$, on the D-Wave Advantage.
- **Initial state:** Best valid FA solution used as the starting state for RA.
- **Quality metric:** Normalized score $\in [0,1]$, standardized by ground truth; improvement = percentage-point gain over FA.

Observation

- ↑ **Problem size** → ↑ **RA improvement,** likely due to higher bit-flip error rates in larger systems**.**
- **Problem type** strongly influences gains → complex or structured problems benefit most from refinement.

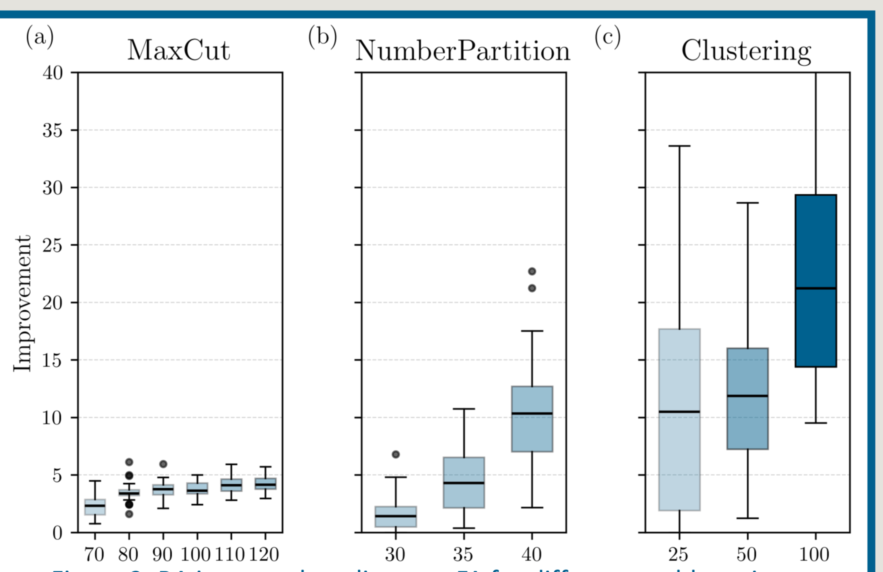


Figure 2: RA improved quality over FA for different problem sizes, darker blue bars indicate larger numbers of logical qubits.

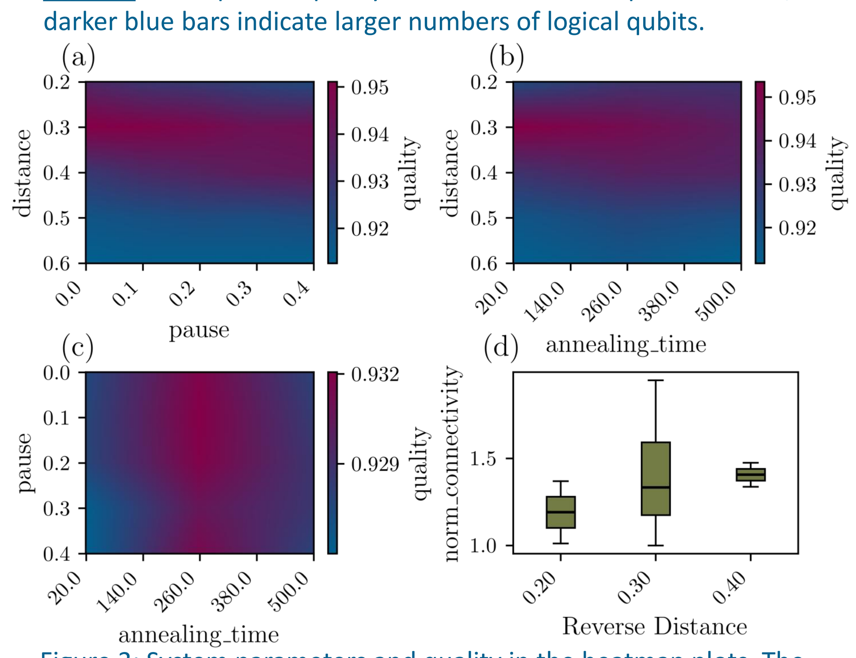


Figure 3: System parameters and quality in the heatmap plots. The box plot shows best performance with respect to connectivity, aggregated over all problems.

Reverse Annealing Parameters

- **Annealing Time vs. Reverse Distance:** Reverse distance is the dominant factor, longer annealing times mainly broaden the viable range of reverse distances.
- **Reverse Distance vs. Pause Duration:** Both increase exploration. However, too long a pause with low reverse distance can cause excessive exploration; the distance effect dominates.
- **Annealing Time vs. Pause Duration:** Mostly independent; only extreme settings (long pause + short anneal) degrade performance.
- **Higher Connectivity:** Highly connected graphs favor larger reverse distances, suggesting a negative correlation between connectivity and optimal exploration.

Freeze-out Point Correlations

- The freeze-out point was approximated using the **slicing method**(slices=distances).
- Across all problem classes, freeze-out occurs around a **reverse distance of 0.3**.
- The relative distance to the final minimum energy serves as an indicator of **system convergence**; smaller values signify that the system has not yet fully settled.

Observation

- **Larger problems** exhibit a more pronounced freeze-out effect, reflecting stronger limitations on late-stage dynamics.
- Among all problem classes, **clustering** shows the most distinct and early freeze-out behavior.

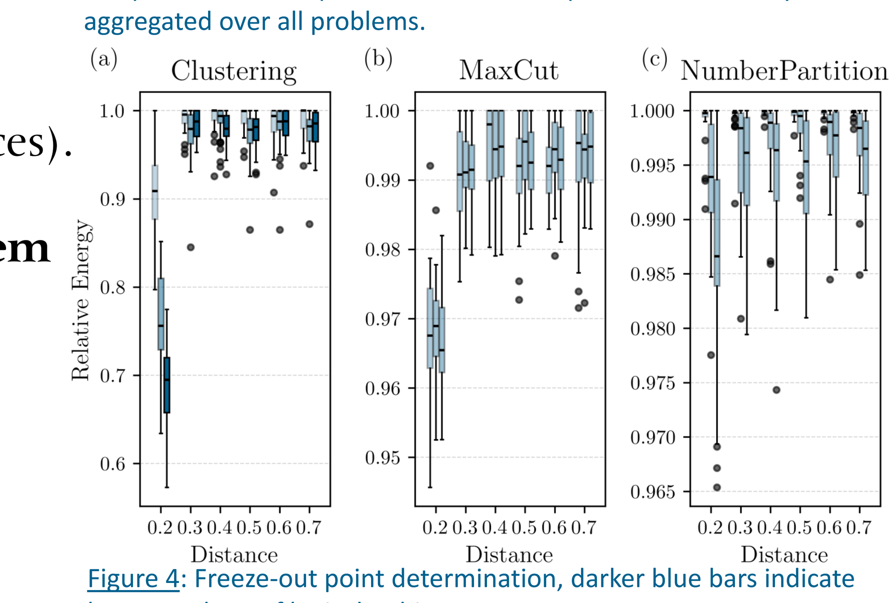


Figure 4: Freeze-out point determination, darker blue bars indicate large numbers of logical qubits.

Conclusion

Reverse Annealing consistently improves upon Forward Annealing when properly tuned, especially for more complex problems. Reverse Distance is the most critical parameter, strongly linked to freeze-out points and problem characteristics. These insights enable a problem-aware tuning strategy to enhance QA performance.

Outlook: Future work will explore whether these patterns generalize across broader problem classes. Understanding the physical mechanisms behind RA's advantages remains an open research direction.

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